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Cosmic Acceleration in Massive Half–Maximal Supergravity

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Abstract

We consider massive half–maximal supergravity in $(d + 3)$ dimensions and compactify it on a symmetric three–space. We find that the static configurations of $\text{Minkowski}_d \times S^3$ obtained by balancing the positive scalar potential for the dilaton and the flux of a three–form through the three–sphere are unstable. The resulting cosmological evolution breaks supersymmetry and leads to an accelerated expansion in d dimensions.

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1 Introduction

Recent cosmological observations seem to indicate that the expansion of the Universe is accelerating. The acceleration of the expansion has been corroborated using the results of the WMAP satellite on the anisotropies of the CMB, the Hubble diagram of type Ia supernovae and the large scale structures of the Universe. This is strong evidence in favour of the existence of a dark energy fluid which may be the realization of a cosmological constant (see e.g. [1] for a recent review). The required energy density of the dark energy fluid is some 120 orders of magnitude below a natural scale such as the Planck mass. A satisfactory explanation for the existence and the smallness of the cosmological constant has not been found yet.

An accelerated phase in the history of the Universe is also advocated to have existed in the early Universe during inflation [2]. Hence the Universe would have undergone at least two phases of accelerated expansion. From the point of view of high energy physics and in particular string theory, the existence of accelerated universes is problematic [3, 4]. First of all, there is no known formulation of string theory in a space-time with a future cosmological event horizon like de Sitter space (see e.g. [5] and [6]). In a nutshell, this springs from the difficulty of formulating S-matrix amplitudes in de Sitter space. Of course, since string theory is valid at very high energy well before the energy scales when the recent acceleration of the expansion of the Universe started, this might not be relevant to the cosmological constant problem. On the contrary, one may hope to describe the acceleration of the expansion within the realm of effective field theories as deduced by compactification of the low energy supergravity theories associated with string theories. At the level of a four dimensional description, natural candidates which may trigger the accelerated expansion can be readily identified with the various moduli arising from the compactification process [7, 8, 9, 10, 11, 12, 13, 14].

Acceleration can arise when a four dimensional theory violates the strong energy condition stating that $p \geq -\rho/3$ where p is the pressure and ρ the energy density. This can be realised with a slow rolling scalar field as well-known in inflation models. However it turns out that string theory and M-theory in 10 and 11 dimensions do not violate the strong energy condition. Upon compactification on a static manifold, the resulting 4d model satisfies the strong energy condition. Therefore no static compactification of string theory or M-theory can lead to an accelerated Universe. This is the Gibbons–Maldacena–Nunez theorem [15, 16]. Of course, one may circumvent the stringent constraint of having a static compactification by allowing a space-time dependence of the breathing mode measuring the size of the internal manifold. In that case, one can deduce that the effective potential for the breathing mode cannot have a stationary point with a positive value of the potential. In particular, runaway potentials of the exponential form [17, 18]

$$V = \Lambda e^{-2c\phi} \tag{1}$$

are allowed. It is well-known that this leads to power law inflation provided [19, 20]

$$c < 1/\sqrt{2}. \tag{2}$$

It turns out the compactifications with fluxes or with an hyperbolic geometry do not lead to $c < 1/\sqrt{2}$. In the same vein, the massive type IIA supergravity compactified on a six torus leads to $c = \sqrt{7}$ [3]. Recently it was conjectured that compactifications leading to $c < 1/\sqrt{2}$ are not allowed in string theory [3]. In the following we will consider a compactification of heterotic string theory on a torus times a circle [21]. The torus plays no role here, the compactification along the circle uses a gauged symmetry of the equations of motion. A consistent truncation of such a $(d+4)$ -dimensional theory results in a massive supergravity theory with a positive potential [21]. Moreover, a scalar field combining the radius of the circle and the dilaton received a positive exponential potential with an exponent $c = \frac{1}{\sqrt{10}} < \frac{1}{\sqrt{2}}$. The resulting theory possesses static solutions in the form of Minkowski space times a 3-sphere traversed by the non-zero flux of a three-form. We show that these static and supersymmetric configurations are unstable. We study the cosmological dynamics of the model and find that cosmological solutions in the form of an accelerating d space-time times a sphere whose size grows with time can be found. This gives an example of a compactification of string theory/supergravity leading to an accelerating Universe.

2 Massive Half-Maximal Supergravity

Here we follow closely the paper [21] where more details can be found. Let us start with $(d+4)$ dimensional half-maximal supergravity, i.e. with sixteen supersymmetries as in heterotic string theory. The bosonic field content is as follows. There is gravity $\hat{g}_{\mu\nu}$, the antisymmetric tensor $\hat{B}_{\mu\nu}$, the dilaton $\hat{\phi}$ and $(6-d)$ vector fields \hat{A}_μ^a . The $(d+4)$ dimensional action reads

$$\mathcal{S}_{d+3} = \int (\hat{R} * 1 - \frac{1}{2} * d\hat{\phi} \wedge d\hat{\phi} - \frac{1}{2} e^{\alpha\hat{\phi}} * \hat{H} \wedge \hat{H} - \frac{1}{2} e^{\frac{1}{2}\alpha\hat{\phi}} * \hat{F}^a \wedge \hat{F}^a). \quad (3)$$

where $*1$ is the volume form and $\alpha^2 = \frac{8}{d+2}$. We have defined $\hat{F}^a = d\hat{A}^a$ and $\hat{H} = d\hat{B} - \frac{1}{2}\hat{F}^a \wedge \hat{A}^a$. The equations of motion are invariant under the two transformations

$$\hat{\phi} \rightarrow \hat{\phi} + \frac{1}{\alpha}\lambda_1, \quad d\hat{s}^2 \rightarrow e^{2\lambda_2} d\hat{s}^2 \quad (4)$$

and

$$\hat{B} \rightarrow e^{-2\lambda_1 + \lambda_2} \hat{B}, \quad \hat{A}^a \rightarrow e^{-\lambda_1 + \lambda_2} \hat{A}^a. \quad (5)$$

The next step consists in dimensionally reducing to $(d+3)$ dimensions around a circle S^1 . This is achieved via the decomposition

$$\begin{aligned} d\hat{s}^2 &= e^{mz} (e^{2\beta\psi} ds^2 + e^{2\gamma\psi} (dz + \tilde{A})^2), & \hat{B} &= B + \tilde{B} \wedge dz \\ \hat{A}^a &= A^a + \xi^a dz, & \hat{\phi} &= \tilde{\phi} + \frac{4}{\alpha} mz, \end{aligned} \quad (6)$$

where z is the coordinate around the circle S^1 and

$$\beta^2 = \frac{1}{2(d+1)(d+2)}, \quad \gamma = -(d+1)\beta. \quad (7)$$

The theory reduces to a half-maximal supergravity theory coupled to a vector multiplet and can be further truncated by putting

$$\tilde{B} = 0, \quad \tilde{A} = 0, \quad \xi^a = 0, \quad A^a = 0, \quad (8)$$

and choosing

$$\psi = -\frac{4\beta}{\alpha}\tilde{\phi}. \quad (9)$$

Redefining now

$$\phi = \frac{2\alpha}{\tilde{a}}\tilde{\phi}, \quad (10)$$

where $\tilde{a} = \sqrt{\frac{8}{d+1}}$, the $(d+3)$ -bosonic dynamics reduce to a field theory whose Lagrangian reads

$$\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{\tilde{a}\phi}H^2 - (d-1)^2m^2e^{-\frac{\tilde{a}}{2}\phi}. \quad (11)$$

First of all, the end result is a massive supergravity theory with a positive potential. Moreover, notice that for $d=4$, the positive potential has an exponent

$$c \equiv \frac{\tilde{a}}{4} = \sqrt{\frac{1}{10}}, \quad d=4, \quad (12)$$

contradicting the conjecture presented in [22]. We will now analyse the dynamics of this theory. It presents an interesting interplay between the potential term and the 3-form term leading to unstable static configurations. These cosmological solutions break supersymmetry.

3 Cosmological spacetimes

From the action, the equations of motion can be found to be

$$\begin{aligned} R_{\mu\nu} &= \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{4}e^{\tilde{a}\phi}\left(H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} - \frac{2}{3(d+1)}H_{(3)}^2g_{\mu\nu}\right) \\ &+ \frac{m^2(d+2)^2}{d+1}e^{-\frac{1}{2}\alpha\phi}g_{\mu\nu}, \end{aligned} \quad (13)$$

$$\square\phi = \frac{e^{\alpha\phi}}{3\sqrt{2(d+1)}}H_{(3)}^2 - \frac{\sqrt{2}(d+2)^2m^2}{\sqrt{d+1}}e^{-\frac{1}{2}\alpha\phi}, \quad (14)$$

$$\nabla^\rho\left(e^{\tilde{a}\phi}H_{\mu\nu\rho}\right) = 0. \quad (15)$$

We will specialise these equations and only consider time dependent solutions, i.e. we are only interested in cosmological solutions. For the metric we choose the ansatz

$$ds^2 = a^2(t)\eta_{ab}dx^a dx^b + b^2(t)g_{ij}dx^i dx^j, \quad (16)$$

where Roman letters at the beginning of the alphabet denote the cosmological d space-time and g_{ij} is a metric of constant curvature k . Here the two scale factors a and b are independent. Notice that the size of the 3-sphere is allowed to vary in time. For the field H we use

$$H_{ijk}(t) = f(t)\epsilon_{ijk}, \quad (17)$$

and zero otherwise. It corresponds to a net flux across the 3-space of curvature k . As long as f is not constant, the flux varies in time. The equations of motion can be found to be

$$-(d-1)\mathcal{H} = \frac{1}{2}\dot{\phi}^2 + \frac{C}{d+1}a^2 - \frac{a^2m^2(d+2)^2}{d+1}e^{-\frac{1}{2}\tilde{a}\phi} \quad (18)$$

$$\frac{\mathcal{H}^2}{a^2} = \frac{1}{2(d-1)(d-2)}\left(\frac{\dot{\phi}}{a}\right)^2 - \frac{C}{d^2-1} + \frac{(d+2)^2}{d^2-1}m^2e^{-\frac{1}{2}\tilde{a}\phi} \quad (19)$$

$$\ddot{\phi} + (d-2)\mathcal{H}\dot{\phi} + 3\frac{\dot{b}}{b}\dot{\phi} = \frac{\sqrt{2}(d+2)^2m^2}{\sqrt{d+1}}a^2e^{-\frac{1}{2}\tilde{a}\phi} - \sqrt{\frac{2}{d+1}}Ca^2 \quad (20)$$

$$\frac{\ddot{b}}{b} + 2\left(\frac{\dot{b}}{b}\right)^2 + (d-2)\mathcal{H}\frac{\dot{b}}{b} + \frac{2ka^2}{b^2} = \frac{a^2}{2}\frac{d-1}{d+1}C + \frac{m^2(d+2)^2}{d+1}a^2e^{-\frac{1}{2}\tilde{a}\phi} \quad (21)$$

in the cosmological context. In these equations, we have defined $\mathcal{H} = a'/a$ (prime denotes derivative with respect to conformal time). The quantity C is defined by

$$f^2(t) = C(t)b(t)^6e^{-\tilde{a}\phi}. \quad (22)$$

Note that this implies, that C is always positive. This ansatz couples the scalar field ϕ , the scale factor b and H_{ijk} in a particular manner. Consistency between the field equations requires that C fullfills

$$\begin{aligned} \frac{d-2}{d^2-1}a^2\dot{C} &= -\frac{3}{d-1}\left(\frac{\dot{b}}{b}\right)\dot{\phi}^2 + \frac{\sqrt{2}(d+2)^2m^2}{(d-1)\sqrt{d+1}}a^2\dot{\phi}e^{-\frac{1}{2}\tilde{a}\phi} \\ &- \frac{\sqrt{2}a^2}{(d-1)\sqrt{d+1}}C\dot{\phi} - \frac{\tilde{a}}{2}\dot{\phi}\left(\frac{(d-2)(d+2)^2}{d^2-1}\right)a^2m^2e^{-\frac{1}{2}\tilde{a}\phi}, \end{aligned} \quad (23)$$

which provides an equation for the evolution of the three-form field. In cosmic time, the equations read

$$H^2 = \frac{1}{2(d-1)(d-2)}\dot{\phi}^2 - \frac{C}{d^2-1} + \frac{(d+2)^2}{d^2-1}m^2e^{-\frac{1}{2}\tilde{a}\phi} \quad (24)$$

$$\ddot{\phi} + (d-1)H\dot{\phi} + 3\frac{\dot{b}}{b}\dot{\phi} = \frac{\sqrt{2}(d+2)^2m^2}{\sqrt{d+1}}e^{-\frac{1}{2}\tilde{a}\phi} - \sqrt{\frac{2}{d+1}}C \quad (25)$$

$$\frac{\ddot{b}}{b} + 2 \left(\frac{\dot{b}}{b} \right)^2 + (d-1)H \frac{\dot{b}}{b} + \frac{2k}{b^2} = \frac{1}{2} \frac{d-1}{d+1} C + \frac{m^2(d+2)^2}{d+1} e^{-\frac{1}{2}\tilde{a}\phi} \quad (26)$$

$$\begin{aligned} \frac{d-2}{d^2-1} \dot{C} &= -\frac{3}{d-1} \left(\frac{\dot{b}}{b} \right) \dot{\phi}^2 + \frac{\sqrt{2}(d+2)^2 m^2}{(d-1)\sqrt{d+1}} \dot{\phi} e^{-\frac{1}{2}\tilde{a}\phi} \\ &\quad - \frac{\sqrt{2}}{(d-1)\sqrt{d+1}} C \dot{\phi} - \frac{\tilde{a}}{2} \dot{\phi} \left(\frac{(d-2)(d+2)^2}{d^2-1} \right) m^2 e^{-\frac{1}{2}\tilde{a}\phi}. \end{aligned} \quad (27)$$

In these equations, a dot denotes a derivative with respect to cosmic time and $H = \dot{a}/a$.

Let us first analyse static solutions. It is easy to see that static backgrounds are given by

$$C_0 = (d+2)^2 m^2 e^{-\frac{\tilde{a}}{2}\phi_0}. \quad (28)$$

Such static configurations are only possible for

$$k = 1, \quad (29)$$

corresponding to a spherical compactification with

$$b_0 = \frac{2}{\sqrt{C_0}}. \quad (30)$$

Moreover the resulting configurations are known to be supersymmetric. Indeed, they are the dimensional reductions of the near horizon limits of $(d+1)$ branes [21].

Let us investigate the stability and consider small (i.e. linear) fluctuations around the static solution above. The fluctuation $\delta\phi$ behaves like a massless field, i.e.

$$(\delta\phi)^{\cdot\cdot} = 0 \quad (31)$$

Hence the field ϕ possesses a flat direction around the static configuration. Similarly the scale factor a fullfills the same type of equation,

$$(\delta a)^{\cdot\cdot} = 0 \quad (32)$$

and the d -dimensional space-times remains static. On the other hand, fluctuations in C are determined by fluctuations in ϕ :

$$\delta C = -\sqrt{\frac{2}{d+1}} m^2 (d+2)^2 e^{-\frac{1}{2}\tilde{a}\phi_0} \delta\phi. \quad (33)$$

From these equations, one can find that fluctuations in b around the static background are governed by

$$(\delta b)^{\cdot\cdot} = \frac{m^2(d+2)^2}{2} e^{-\frac{1}{2}\tilde{a}\phi_0} \left[\delta b - \sqrt{\frac{2}{5}} \delta\phi \right], \quad (34)$$

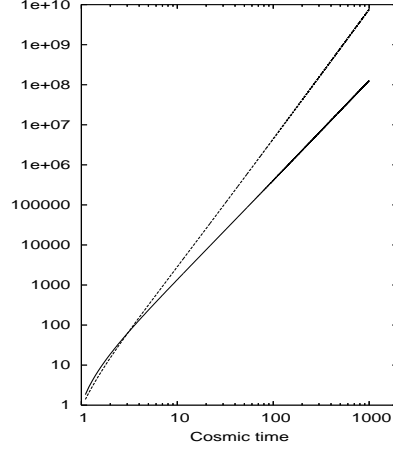


Figure 1: Evolution of the scale factors $a(t)$ (solid line) and $b(t)$ (dashed line) as a function of cosmic time t . In this example we have chosen $m = 6$. The initial conditions for ϕ and C are given by $\phi_{in} = 1.0$ and $C_{in} = 10$ (in natural units).

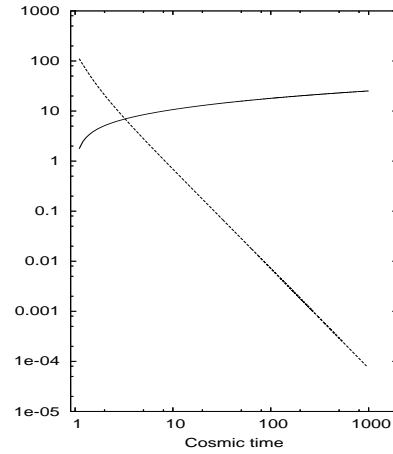


Figure 2: Evolution of $\phi(t)$ (solid line) and $C(t)$ (dashed line) as a function of cosmic time t . The parameters and initial conditions are chosen as in Fig. 1.

which signals an exponential instability for b , sourced by the field fluctuation $\delta\phi$. These considerations tell us, that fluctuations quickly become non-linear and therefore the sub-space defined by the metric $\tilde{g}_{ij} = b(t)g_{ij}$ is unstable. To go beyond the linear perturbation analysis, we resort to a numerical study in which we focus on $d = 4$.

A typical example is given in Figure 1 and Figure 2. The scale factor b grows faster than the scale factor a and C decays. This holds also for other initial conditions and parameters than those used in Figure 1 and Figure 2. The effects of m , C_0 and ϕ_0 are such they only affect the initial behaviour of a , b , C and ϕ . However, after a certain amount of time the behaviour of the fields is similar to the one showed in the Figures. In particular, it is noticeable that the size of the sphere grows with time and is not bounded from above. In the future the size of the sphere becomes of the order of the Hubble radius of the 4d space-time implying that, in a sense, space-time decompactifies.

This is not the only peculiar feature of the model. Let us now turn to the physics as seen by test matter. Let us consider that matter is only present in 4d, and therefore corresponds to an action

$$S_m = \int d^4x \mathcal{L}_m(\psi_m, g_{ab}), \quad (35)$$

where ψ_m is a 4d matter field coupled to the metric $g_{ab} = a^2\eta_{ab}$. Let us now consider the Einstein-Hilbert term after dimensional reduction and integration over the 3-sphere

$$\int d^7x R \supset \int d^4x b^3 R_{(4)}, \quad (36)$$

where the volume of the 3-sphere has been normalised to unity and $R_{(4)}$ is the curvature of the metric g_{ab} . Now the gravitational constant is time dependent, involving a b^3 factor. One can go to the Einstein frame by defining

$$g_{ab}^E = b^3 g_{ab}. \quad (37)$$

In this frame, Newton's constant is time independent while the coupling to matter reads

$$S_m = \int d^4x \mathcal{L}_m(\psi_m, b^{-3} g_{ab}^E). \quad (38)$$

In particular, the effective scale factor is

$$a_E = b^{3/2} a. \quad (39)$$

which grows faster than t , i.e. leads to an accelerated expansion (see Figure 3).

In the low-energy effective theory, the coupling to matter is not minimal anymore but involves the factor b^{-3} . In the Einstein frame, particles move no longer on geodesics and their masses are no longer constant: massive particles of mass m_G are subject to a force $F_\mu = -m_G \partial_\mu \ln b^{-3/2}$ whose only non-vanishing component is

$$F_0 = \frac{3}{2} m_G \mathcal{H}_b, \quad (40)$$

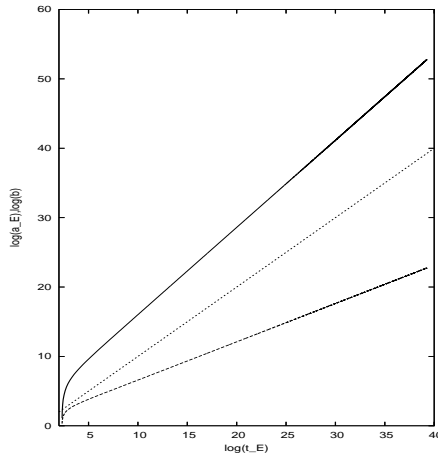


Figure 3: Evolution of the Einstein frame scale factor a_E (solid line) and b (long dashed line) as a function of time in the Einstein frame t_E . The parameters and initial conditions are chosen as in Fig. 1. The short dashed line shows the line in which $a_E \propto t_E$. Thus, in the effective four-dimensional theory the scale factor grows faster than t_E , i.e. the universe is accelerating. The scale factor b grows slower than t_E .

where $\mathcal{H}_b = \frac{\dot{b}}{b}$. The time variation of masses is given by

$$\frac{\dot{m}_G}{m_G} = -3\mathcal{H}_b. \quad (41)$$

This seems to result in a very large variation which might prevent the use of the model for late time acceleration. To really answer this question one needs to take into account the back-reaction of matter on the expansion of the Universe. This, however, is beyond the scope of this paper and left for future work.

4 Conclusions

We have seen in this paper that the supergravity theory presented in [21] can lead to an accelerated expansion of the universe. Although the theory presented is not consistent with standard cosmology, as it predicts a huge variation of masses during cosmic history, it might be a starting point for future investigations. For example, we have not studied brane sources in the theory as well as more complicated compactification schemes. In the present case, the cosmological solutions and the instability of the supersymmetric configurations result from the required fine-tuning to obtain a static and supersymmetric solution when compactifying on a three-sphere. The flux through the compact three-sphere can only compensate the positive and run-away potential for the dilaton when the flux is fine-tuned. Away from this fine-tuned value, the system is unstable and the dilaton starts

running under the influence of the breathing mode measuring the size of the three-sphere. The fact that the instability is triggered by the breathing mode is in accord with a violation of one of the premises of the Gibbons-Maldacena–Nunez theorem, i.e. the requirement of a static compactification. It remains to be seen whether such a model may have some phenomenological applications in the presence of matter.

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